http://CursoDeCalculo.com
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Integrales Directas y por Sustitución

Sección ✳1 Funciones algebraicas

$$\int 3ay\,dy = 3a\int y\,dy = (3a)\left(\frac{y^{2+1}}{2+1}\right) + C = (3a)\left(\frac{y^3}{3}\right) + C = (a)(y^3) + C = ay^3 + C$$

$$\int \left(1 + \frac{2}{x^2} + \frac{3}{x^3}\right) dx = \int dx + 2\int x^{-2}\,dx + 3\int x^{-3}\,dx$$

$$= x + \frac{2x^{-2+1}}{-2+1} + \frac{3x^{-3+1}}{-3+1} + C = x + \frac{2x^{-1}}{-1} + \frac{3x^{-2}}{-2} = x - \frac{2}{x} - \frac{3}{2x^2} + C$$

$$\int (x+1)(3x-2)\,dx = \int (3x^2 + x - 2)\,dx = 3\int x^2\,dx + \int x\,dx - 2\int dx$$

$$= \frac{3x^{2+1}}{2+1} + \frac{x^2}{2} - 2x + C = \frac{3x^3}{3} + \frac{x^2}{2} - 2x + C = x^3 + \frac{x^2}{2} - 2x + C$$

$$\int (t^2+1)^2\,dt = \int (t^4 + 2t^2 + 1)\,dt = \int t^4\,dt + 2\int t^2\,dt + \int dt$$

$$= \frac{t^{4+1}}{4+1} + \frac{2t^{2+1}}{2+1} + t + C = \frac{t^5}{5} + \frac{2t^3}{3} + t + C$$

$$\int \left(\frac{y}{4} - 3\right)^2 dy = \int \left(\frac{y^2}{4} - \frac{3}{2}y + 9\right) dy = \frac{1}{4}\int y^2\,dy - \frac{3}{2}\int y\,dy + 9\int dy$$

$$= \frac{y^{2+1}}{4(2+1)} - \frac{3y^2}{2(2)} + 9y + C = \frac{y^3}{12} - \frac{3y^2}{4} + 9y + C$$

$$\int y(a + by^2)^{-2}\,dy$$

$$u = a + by^2, \qquad du = 2bydy, \qquad ydy = \frac{du}{2b}$$

$$\int y(a+by^2)^{-2}\,dy = \frac{1}{2b}\int u^{-2}du = \frac{u^{-2+1}}{2b(-2+1)} + C = \frac{u^{-1}}{2b(-1)} + C$$

$$= -\frac{1}{2bu} + C = -\frac{1}{2b(a+by^2)} + C$$

$$\int \sqrt{2y}\,dy = \sqrt{2}\int \sqrt{y}dy = \sqrt{2}\int y^{\frac{1}{2}}\,dy = \frac{\sqrt{2}y^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{\sqrt{2}y^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{3\sqrt{2}y^{\frac{3}{2}}}{2} + C$$

$$= \frac{3\sqrt{2}\sqrt{y^3}}{2} + C = \frac{3\sqrt{2y^3}}{2} + C$$

$$\int x\sqrt{4x^2+3}dx$$

$$u = 4x^2 + 3, \qquad du = 8xdx, \qquad xdx = \frac{du}{8}$$

$$\int x\sqrt{4x^2+3}dx = \frac{1}{8}\int \sqrt{u}\,du = \frac{1}{8}\int u^{\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}+1}}{8\left(\frac{1}{2}+1\right)} + C = \frac{u^{\frac{3}{2}}}{8\left(\frac{3}{2}\right)} + C$$

$$= \frac{\sqrt{u^3}}{12} + C = \frac{\sqrt{(4x^2+3)^3}}{12} + C$$

$$\int t\sqrt{2t^2+3}dt$$

$$u = 2t^2 + 3, \qquad du = 4tdt, \qquad tdt = \frac{du}{4}$$

$$\int t\sqrt{2t^2+3}dt = \frac{1}{4}\int \sqrt{u}\,du = \frac{1}{4}\int u^{\frac{1}{2}}\,du = \frac{u^{\frac{1}{2}+1}}{4\left(\frac{1}{2}+1\right)} + C = \frac{u^{\frac{3}{2}}}{4\left(\frac{3}{2}\right)} + C$$

$$= \frac{\sqrt{u^3}}{6} + C = \frac{\sqrt{(2t^2+3)^3}}{6} + C$$

$$\int \left(\frac{1 - x + 2x^3}{x^3}\right) dx = \int (x^{-3} - x^{-2} + 2)\, dx = \int x^{-3}\, dx - \int x^{-2}\, dx + 2\int dx$$

$$= \frac{x^{-3+1}}{-3+1} - \frac{x^{-2+1}}{-2+1} + 2x + C = \frac{x^{-2}}{-2} - \frac{x^{-1}}{-1} + 2x + C = -\frac{1}{2x^2} + \frac{1}{x} + 2x + C$$

$$\int \left(\frac{4x^2 - 2\sqrt{x}}{x}\right) dx = \int \left(4x - \frac{2x^{\frac{1}{2}}}{x}\right) dx = \int \left(4x - 2x^{-\frac{1}{2}}\right) dx$$

$$= 4\int x\, dx - 2\int x^{-\frac{1}{2}}\, dx = \frac{4x^2}{2} - \frac{2x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = 2x^2 - \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + C = 2x^2 - 4\sqrt{x} + C$$

$$\int \sqrt{25 - 9x^2}\, dx$$

$$u = 3x, \qquad u^2 = 9x^2, \qquad a = 5, \qquad a^2 = 25, \qquad du = 3dx, \qquad dx = \frac{du}{3}$$

$$\int \sqrt{25 - 9x^2} dx = \frac{1}{3}\int \sqrt{a^2 - u^2} du = \frac{1}{3}\left(\frac{u}{2}\sqrt{a^2 - u^2} + \frac{a^2}{2}\ sen^{-1}\frac{u}{a}\right) + C$$

$$= \frac{1}{3}\left(\frac{3x}{2}\sqrt{25 - 9x^2} + \frac{25}{2}\ sen^{-1}\frac{3x}{5}\right) + C = \frac{x}{2}\sqrt{25 - 9x^2} + \frac{25}{6}\ sen^{-1}\frac{3x}{5} + C$$