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Profesor Raúl Vega Muñoz

Integrales por Sustitución (Cambio de Variable)

Sección ✳1 Funciones algebraicas, trigonométricas y logarítmicas

✳40 ---
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$$\int \sqrt{m+ny}\,dy = \frac{1}{n}\int \sqrt{m+ny}\;n\,dy = \frac{1}{n}\int \sqrt{u}\,du = \frac{1}{n}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{1}{n}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3n}u^{\frac{3}{2}} + C$$

$$= \frac{2}{3n}\sqrt{u^3} + C = \frac{2}{3n}\sqrt{(m+ny)^3} + C$$

$u = m + ny$
$du = n\,dy$

✳41 ---
-

$$\int \sqrt{5x-3}\,dx = \frac{1}{5}\int \sqrt{5x-3}\;5\,dx = \frac{1}{5}\int \sqrt{u}\;du = \frac{1}{5}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C = \frac{1}{5}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{15}u^{\frac{3}{2}} + C = \frac{2}{15}\sqrt{u^3} + C$$

$$= \frac{2}{15}\sqrt{(5x-3)^3} + C$$

$u = 5x - 3$
$du = 5\,dx$

✳42 ---
-

$$\int \frac{t\,dt}{\sqrt{at^2+b}} = \frac{1}{2a}\int \frac{2at\,dt}{\sqrt{at^2+b}} = \frac{1}{2a}\int \frac{du}{\sqrt{u}} = \frac{1}{2a}\int u^{-\frac{1}{2}}du = \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{u} + C = 2\sqrt{at^2+b} + C$$

$u = at^2 + b$
$du = 2at\ dt$

✳43 --
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$$\int \frac{dx}{\sqrt[3]{9x-1}} = \frac{1}{9}\int \frac{9dx}{\sqrt[3]{9x-1}} = \frac{1}{9}\int \frac{du}{\sqrt[3]{u}} = \frac{1}{9}\int u^{-\frac{1}{3}}du = \frac{1}{9}\cdot\frac{u^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + C = \frac{1}{9}\cdot\frac{u^{\frac{2}{3}}}{\frac{2}{3}} + C$$

$$= \frac{3}{18}u^{\frac{2}{3}} + C = \frac{1}{6}\sqrt[3]{u^2} + C = \frac{1}{6}\sqrt[3]{(9x-1)^2} + C$$

$u = 9x - 1$
$du = 9dx$

✳44 --
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$$\int \left(\sqrt{x} - 4\right)^2 dx = \int \left(x - 8x^{\frac{1}{2}} + 16\right) dx = \int x\ dx - 8\int x^{\frac{1}{2}}\ dx + 16\int dx =$$

$$= \frac{x^2}{2} - \frac{8x^{\frac{3}{2}}}{\frac{3}{2}} + 16x + C = \frac{x^2}{2} - \frac{16\sqrt{x^3}}{3} + 16x + C$$

✳45 --
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$$\int \frac{x\ dx}{(3x^2-4)^4} = \frac{1}{6}\int \frac{6x\ dx}{(3x^2-4)^4} = \frac{1}{6}\int \frac{du}{u^4} = \frac{1}{6}\int u^{-4}du = \frac{1}{6}\cdot\frac{u^{-4+1}}{-4+1} + C =$$

$$= \frac{1}{6}\cdot\frac{u^{-3}}{-3} + C = -\frac{1}{18u^3} + C = -\frac{1}{18(3x^2-4)^3} + C$$

$u = 3x^2 - 4$
$du = 6x\ dx$

✳46 --
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$$\int \frac{5\,dx}{(3x-4)^2} = \frac{5}{3}\int \frac{\frac{3}{5}\cdot 5\,dx}{(3x-4)^2} = \frac{5}{3}\int \frac{3\,dx}{(3x-4)^2} = \frac{5}{3}\int \frac{du}{u^2} = \frac{5}{3}\cdot\frac{u^{-2+1}}{-2+1} + C$$

$$= \frac{5}{3}\cdot\frac{u^{-1}}{-1} + C = -\frac{5}{3u} + C = -\frac{5}{3(3x-4)} + C$$

$u = 3x - 4$
$du = 3\,dx$

✳47 ---
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$$\int \frac{8x\,dx}{(2x^2+5)^4} = 2\int \frac{4x\,dx}{(2x^2+5)^4} = 2\int \frac{du}{(u)^4} = 2\int u^{-4}du = \frac{2u^{-4+1}}{-4+1} + C$$

$$= \frac{2u^{-3}}{-3} + C = -\frac{2}{3u^3} + C = -\frac{2}{3(2x^2+5)^3} + C$$

$u = 2x^2 + 5$
$du = 4x\,dx$

✳48 ---
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$$\int \frac{\left(\sqrt{x}-b\right)^2}{\sqrt{x}}\,dx = 2\int \frac{\left(\sqrt{x}-b\right)^2}{2\sqrt{x}}\,dx = 2\int u^2du = \frac{2u^3}{3} + C = \frac{2(\sqrt{x}-b)^3}{3} + C$$

$u = \sqrt{x} - b$

$du = \frac{1}{2\sqrt{x}}dx$

✳49 ---
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$$\int \frac{dt}{at+b} = \frac{1}{a}\int \frac{a\,dt}{at+b} = \frac{1}{a}\int \frac{du}{u} = \frac{1}{a}\ln u + C = \frac{1}{a}\ln|at+b| + C$$

$u = at + b$
$du = a\,dt$

✳50 ---
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$$\int \frac{x\,dx}{3x^2-4} = \frac{1}{6}\int \frac{6x\,dx}{3x^2-4} = \frac{1}{6}\int \frac{du}{u} = \frac{1}{6}\ln u + C = \frac{1}{6}\ln|3x^2-4| + C$$

$u = 3x^2 - 4$
$du = 6x\,dx$

✳51 ---
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$$\int \frac{dx}{x+3} = \int \frac{du}{u} = \ln u + C = \ln|x+3| + C$$

$u = x + 3$
$du = dx$

✳52 ---
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$$\int \frac{4x\,dx}{2x^2-6} = \int \frac{du}{u} = \ln u + C = \ln|2x^2-6| + C$$

$u = 2x^2 - 6$
$du = 4x\,dx$

✳53 ---
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$$\int \frac{(2x-3)dx}{(x^2-3x+6)^2} = \int \frac{du}{u^2} = \frac{u^{-2+1}}{-2+1} + C = -\frac{1}{u} + C = -\frac{1}{x^2-3x+6} + C$$

$u = x^2 - 3x + 6$
$du = (2x-3)dx$

✳54 ---
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$$\int (x^2-2)\sqrt{x^3-6x+3}dx = \frac{1}{3}\int 3(x^2-2)\sqrt{x^3-6x+3}dx = \frac{1}{3}\int \sqrt{u}\,du = \frac{1}{3}\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C =$$

$$\frac{1}{3}\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{2}{9}\sqrt{u^3}+C=\frac{2}{9}\sqrt{(x^3-6x+3)^3}+C$$

$u=x^3-6x+3$
$du=(3x^2-6)dx$

✳55 --
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$$\int\frac{y^{n-1}dy}{(ay^n+b)^m}=\frac{1}{na}\int\frac{nay^{n-1}dy}{(ay^n+b)^m}=\frac{1}{na}\int\frac{du}{u^m}=\frac{1}{na}\int u^{-m}du=\frac{1}{na}\cdot\frac{u^{1-m}}{1-m}+C=\frac{(ay^n+b)^{1-m}}{na-nam}+C$$

$u=ay^n+b$
$du=nay^{n-1}dy$

✳56 --
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$$\int e^{3x}(1-e^{3x})^2dx=-\frac{1}{3}\int -3e^{3x}(1-e^{3x})^2dx=-\frac{1}{3}\int u^2du=-\frac{1}{3}\cdot\frac{u^{2+1}}{2+1}+C$$

$$=-\frac{1}{3}\cdot\frac{u^3}{3}+C=-\frac{1}{9}(1-e^{3x})^3+C$$

$u=1-e^{3x}$
$du=-3e^{3x}dx$

✳57 --
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$$\int\frac{(4-In|x+3|)^3\,dx}{x+3}=\int u^3du=\frac{u^{3+1}}{3+1}+C=\frac{u^4}{4}+C=\frac{(4-In|x+3|)^4}{4}+C$$

$u=4-In|x+3|$

$$du=\frac{dx}{x+3}$$

✳58 --
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$$\int \cos 4x\,(1-sen\,4x)^3dx = -\frac{1}{4}\int -4\cos 4x\,(1-sen\,4x)^3dx = -\frac{1}{4}\int u^3\,du = -\frac{1}{4}\cdot\frac{u^{3+1}}{3+1}+C$$

$$= -\frac{1}{16}u^4+C = -\frac{1}{16}(1-sen\,4x)^4+C$$

$u = 1 - sen\,4x$
$du = -4cos\,4x\,dx$

✳59 --
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$$\int \csc^2 x\sqrt{3+\cot x}\;dx = \int \sqrt{u}\,du = \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C = \frac{u^{\frac{3}{2}}}{\frac{3}{2}}+C = \frac{2}{3}\sqrt{u^3}+C = \frac{2}{3}\sqrt{(3+\cot x\,)^3}+C$$

$u = 3 + \cot x$
$du = -\csc^2 x\;dx$

✳60 --
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$$\int \frac{\sec 2x\tan 2x}{\sqrt{1-\sec 2x}}dx = -\frac{1}{2}\int \frac{-2\sec 2x\tan 2x}{\sqrt{1-\sec 2x}}dx = -\frac{1}{2}\int u^{-\frac{1}{2}}du = -\frac{1}{2}\cdot\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C$$

$$= -\frac{1}{2}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}}+C = -\sqrt{u}+C = -\sqrt{1-\sec 2x}+C$$

$u = 1 - \sec 2x$
$du = -2\sec 2x\tan 2x\,dx$

✳61 --
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$$\int \frac{\cos ax}{1-sen\,ax}\;dx = -\frac{1}{a}\int \frac{-a\cos ax}{1-sen\,ax}\;dx = -\frac{1}{a}\int \frac{du}{u} = -\frac{1}{a}\ln|u|+C = -\frac{1}{a}\ln|1-sen\,ax|+C$$

$u = 1 - sen\,ax$
$du = -a\,cos\,ax\,dx$

✳62 --
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$$\int \frac{e^{\sqrt{x}}\sqrt{e^{\sqrt{x}}-1}}{\sqrt{x}}\,dx = 2\int \frac{e^{\sqrt{x}}\sqrt{e^{\sqrt{x}}-1}}{2\sqrt{x}}\,dx = 2\int \sqrt{u}\,du = \frac{2u^{\left(\frac{1}{2}+1\right)}}{\frac{1}{2}+1} + C = \frac{2u^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{4}{3}\sqrt{u^3} + C$$

$$= \frac{4}{3}\sqrt{(e^{\sqrt{x}}-1)^3} + C$$

$u = e^{\sqrt{x}} - 1$
$du = e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} dx$

✳63 --
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$$\int \cot x\,(2 + In\,|sen\,x|)dx = \int u\,du = \frac{u^2}{2} + C = \frac{(\,2 + In\,|sen\,x|)^2}{2} + C$$

$u = 2 + In\,|sen\,x|$
$du = \cot x\,dx$

✳64 --
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$$\int \frac{sen\,2x\,dx}{(1-\cos^2 x)^3} = 2\int \frac{sen\,x\cos x\;dx}{(sen^2 x)^3} = 2\int \frac{\cos x\;dx}{sen^5\,x} = 2\int u^{-5}\,du = \frac{2u^{-5+1}}{-5+1} + C = \frac{2u^{-4}}{-4} + C$$

$$= -\frac{1}{2u} + C = -\frac{1}{2\,sen\,x} + C = -\frac{1}{2}\csc x + C$$

$sen^2 x = 1 - \cos x$
$sen\,2x = 2\,sen\,x\cos x$
$u = sen\,x$
$du = \cos x\;dx$

✳65 --
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$$\int sen^2 bx\cos bx\;dx = \frac{1}{b}\int sen^2 bx \cdot b\cos bx\,dx = \frac{1}{b}\int u\,du = \frac{u^2}{2b} + C = \frac{sen^2 bx}{2b} + C$$

$u = \ sen\ bx$
$du = b \cos bx \ dx$

✳66 ---
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$$\int \cot mx \csc^2 mx \, dx = -\frac{1}{m}\int -m \ \cot mx \csc^2 mx \, dx = -\frac{1}{m}\int u \, du = -\frac{u^2}{2m} + C = -\frac{cot^2 mx}{2m} + C$$

$u = \cot x$
$du = \ -m \ \csc^2 mx \ \ dx$

✳67 ---
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$$\int \cos^2 4x \ sen\ 4x \, dx = -\frac{1}{4}\int -4\cos^2 4x \ \ sen\ 4x \, dx = -\frac{1}{4}\int u^2 \, du = -\frac{1}{4}\cdot\frac{u^{2+1}}{2+1} + C = -\frac{1}{12}u^3 + C$$

$$= -\frac{1}{12}cos^3 4x + C$$

$u = \cos 4x$

$du = \ -4 \ sen\ 4x \ dx$

✳68 ---
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$$\int \frac{\cos 5x}{\sqrt{sen\ 5x + 4}} dx = \frac{1}{5}\int \frac{5\cos 5x}{\sqrt{sen\ 5x + 4}} dx = \frac{1}{5}\int u^{-\frac{1}{2}} du = \frac{1}{5}\cdot\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{5}\sqrt{u} + C = \frac{2}{5}\sqrt{sen\ 5x + 4} + C$$

$u = \ sen\ 5x + 4$

$du = 5 \ \cos 5x \ \ dx$

✳69 ---
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$$\int \frac{4x+2}{x+2} dx = 4\int \frac{x+\frac{1}{2}}{x+2} dx = 4\int \frac{(x+2)-\frac{3}{2}}{x+2} dx = 4\int dx - 6\int \frac{dx}{x+2} = 4x - 6\ln|x+2| + C$$

$u = x + 2$
$du = dx$

✳70 --
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$$\int \frac{(3x^2+2)dx}{x-1} = \int \left(3x + 3 + \frac{5}{x-1}\right) dx = 3\int x\,dx + 3\int dx + 5\int \frac{dx}{x-1}$$

$$= \frac{3x^2}{2} + 3x + 5\ln|x-1| + C$$

$u = x - 1$
$du = dx$

✳71 --
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$$\int \frac{dy}{y\,In^2\,y} = \int u^{-2}\,du = \frac{u^{-2+1}}{-2+1} + C = -\frac{1}{u} + C = -\frac{1}{\ln y} + C$$

$u = \ln y$
$du = \frac{1}{y}dy$

✳72 --
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$$\int \frac{dx}{2x\,In\,3x} = \frac{1}{2}\int \frac{dx}{x(\ln 3 + \ln x)} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln u + C = \frac{1}{2}\ln|\ln 3x| + C$$

$\ln 3x = \ln 3 + \ln x$

$u = \ln 3 + \ln x$
$du = \frac{1}{x}dx$

✳73 --
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$$\int x^n\sqrt{ax^{n+1}+b}\;dx=\frac{1}{a(n+1)}\int a(n+1)x^n\sqrt{ax^{n+1}+b}\;dx=\frac{1}{a(n+1)}\int u^{\frac{1}{2}}du=$$

$$=\frac{1}{a(n+1)}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{a(n+1)}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+C=\frac{2\sqrt{u^3}}{3a(n+1)}+C=\frac{2\sqrt{(ax^{n+1}+b)^3}}{3a(n+1)}+C$$

$u=\ ax^{n+1}+b$
$du=a(n+1)x^n dx$

✳74 ---
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$$\int\frac{1}{x^3}\sqrt{1-\frac{1}{x^2}}\;dx=\frac{1}{2}\int\frac{2}{x^3}\sqrt{1-\frac{1}{x^2}}\;dx=\frac{1}{2}\int u^{\frac{1}{2}}du=\frac{1}{2}\cdot\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}+C=\frac{1}{2}\cdot\frac{u^{\frac{3}{2}}}{\frac{3}{2}}+C$$

$$=\frac{1}{3}\sqrt{u^3}+C=\frac{1}{3}\sqrt{\left(1-\frac{1}{x^2}\right)^3}+C$$

$u=\ 1-\dfrac{1}{x^2}$
$du=\ 2x^{-3}dx$

✳75 ---
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$$\int\csc^2 3x\cos 3x\;dx=\int\frac{\cos 3x}{sen^2 3x}dx=\frac{1}{3}\int\frac{3\cos 3x}{sen^2 3x}dx=\frac{1}{3}\int u^{-2}du=\frac{1}{3}\left(\frac{u^{-2+1}}{-2+1}\right)+C=\frac{1}{3}\left(\frac{u^{-1}}{-1}\right)+C$$

$$=-\frac{1}{3u}+C=-\frac{1}{3\;sen\;3x}+C$$

$u=\ sen\;3x$
$du=3\cos 3x\;dx$

✳76 ---
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$$\int\left(\frac{2}{(x+1)^3}-\frac{3}{(x+1)^2}+\frac{4}{x+1}\right)dx=2\int\frac{dx}{(x+1)^3}-3\int\frac{dx}{(x+1)^2}+4\int\frac{dx}{(x+1)}$$

$$= 2\int u^{-3}\,du - 3\int u^{-2}\,du + 4\int \frac{du}{u} = \frac{2u^{-2}}{-2} - \frac{3u^{-1}}{-1} + 4\ln|u| + C$$

$$= -\frac{1}{u^2} + \frac{3}{u} + 4\ln|u| + C = -\frac{1}{(x+1)^2} + \frac{3}{x+1} + 4\ln|x+1| + C$$

$u = x + 1$
$du = dx$

*77 --
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$$\int \left(\frac{3}{x+2} - \frac{4}{x+5}\right) dx = 3\int \frac{dx}{x+2} - 4\int \frac{dx}{x+5} = 3\ln|x+2| - 4\ln|x+5| + C$$

*78 --
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$$\int \left(\frac{3}{2x-1} + \frac{5}{3x-4}\right) dx = 3\int \frac{dx}{2x-1} + 5\int \frac{dx}{3x-4} = \frac{3}{2}\int \frac{2dx}{2x-1} + \frac{5}{3}\int \frac{3dx}{3x-4} = \frac{3}{2}\ln|2x-1| + \frac{5}{3}\ln|3x-4| + C$$

$u = 2x - 1$
$du = 2\,dx$
$v = 3x - 4$
$du = 3\,dx$

*79 --
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$$\int \frac{sen\,x}{\sqrt[3]{\cos^2 x}}\,dx = -\int \frac{-\,sen\,x}{\sqrt[3]{\cos^2 x}}\,dx = -\int u^{-\frac{2}{3}}\,du = -\frac{u^{-\frac{2}{3}+1}}{-\frac{2}{3}+1} + C = -\frac{u^{\frac{1}{3}}}{\frac{1}{3}} + C = -3\sqrt[3]{u} + C = -3\sqrt[3]{\cos x} + C$$

$u = \cos x$
$du = -sen\,x\,dx$

*80 --
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$$\int sen^3x\, sen\, 2x\, dx = \int sen^3x \cdot 2\, sen\, x \cos x\; dx = 2\int sen^4x \cdot \cos x\; dx = 2\int u^4\, du = \frac{2u^5}{5} + C = \frac{2sen^5x}{5} + C$$

$sen\, 2x = 2\, sen\, x \cos x$
$u = sen\, x$
$du = \cos x\; dx$

✳81 ---
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$$\int \frac{dw}{sen^2\sqrt{1 - cotw}} = \int u^{-\frac{1}{2}}\, du = \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{u} + C = 2\sqrt{1 - cotw} + C$$

$\frac{1}{sen^2w} = \csc^2 w$
$u = 1 - cotw$
$du = \csc^2 w\; dw$

✳82 ---
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$$\int \frac{3\, sen\, y \cos y}{\sqrt{1 - 2sen^2\, y}} = 3\int \frac{sen\, y \cos y}{\sqrt{1 - 2sen^2\, y}} = \frac{3}{2}\int \frac{2sen\, y \cos y}{\sqrt{1 - 2sen^2\, y}} = \frac{3}{2}\int \frac{sen\, 2y}{\sqrt{\cos 2y}} = -\frac{3}{4}\int \frac{-2\, sen\, 2y}{\sqrt{\cos 2y}} = -\frac{3}{4}\int u^{-\frac{1}{2}}du$$

$$= -\frac{3}{4}\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = -\frac{3}{4}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = -\frac{3}{2}\sqrt{u} + C = -\frac{3}{2}\sqrt{\cos 2y} + C$$

$1 - 2sen^2\, y = \cos 2y$
$2\, sen\, y \cos y = sen\, 2y$

$u = \cos 2y$
$du = -2sen\, 2y$

✳83 ---
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$$\int \sqrt{1 + \cos x}\; dx = \int \sqrt{1 + \cos x} \cdot \frac{\sqrt{1 - \cos x}}{\sqrt{1 - \cos x}}\; dx = \int \frac{\sqrt{1 - \cos^2 x}}{\sqrt{1 - \cos x}}\; dx = \int \frac{\sqrt{sen^2 x}}{\sqrt{1 - \cos x}}\; dx = \int \frac{sen\, x}{\sqrt{1 - \cos x}}\; dx$$

$$= \int u^{-\frac{1}{2}} du = \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + C = 2\sqrt{u} + C = 2\sqrt{1-\cos x} + C$$

$u = 1 - \cos x$
$du = sen\, x\, dx$

✳84 --
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$$\int \frac{sen^{\frac{3}{4}}x}{\cos^{\frac{11}{4}} x} dx = \int \frac{sen^{\frac{3}{4}}x}{\cos^{\frac{3}{4}} x \cdot \cos^2 x} dx = \int \sqrt[4]{\tan^3 x} \cdot \sec^2 x\, dx = \int u^{\frac{3}{4}} du = \frac{u^{\frac{3}{4}+1}}{\frac{3}{4}+1} + C = \frac{u^{\frac{7}{4}}}{\frac{7}{4}} + C$$

$$= \frac{4}{7}\sqrt[4]{u^7} + C = \frac{4}{7}\sqrt[4]{tan^7 x} + C$$

$u = \tan x$
$du = \sec^2 x\, dx$